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Application of Lopida law in the problem of high school inequality persistence

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Abstract:In the context of the new college entrance examination, there are stricter requirements for the academic assessment of middle school students, especially for regions such as Shandong where the scores are relatively high and independent proposition is required, there will be a phenomenon of high difficulty in the college entrance examination papers. Mathematics, a subject with high scores and difficulty, becomes more important. In order to cope with this situation, in daily mathematics teaching, we should not be limited to textbook knowledge, but should expand some knowledge of extracurricular or college advanced mathematics, so that middle school students can better cope with the new college entrance examination questions. This article aims to explore the application of the Lopida's rule in the problem of inequality constancy in high school mathematics. Through a deep understanding and analysis of the Lopida rule, combined with practical cases of the new college entrance examination mathematical inequality problem after reform, it can be proven that the Lopida rule is effective and practical in solving such problems.

Key Words: l'hospital rule; High school mathematics; Inequality; The problem of constant establishment;

1. Introduction

In the study of high school mathematics, the problem of inequality constant establishment has always been one of the difficulties that students feel confused and difficult to solve. This kind of problem often involves the complex relationship of multiple variables, which needs to be solved with a variety of mathematical knowledge and skills. As an effective tool to solve the limit problem, Lopida's law is widely and deeply used in the field of mathematics. However, in the teaching of high school mathematics, the application of the law is often ignored or only stays on the surface. Therefore, this paper aims to explore the application of Lopida's law in the problem of inequality persistence in high school mathematics, so as to provide new ideas and methods for the teaching and learning of high school mathematics.

The basic idea of Lopida's law is to solve the limit problem by seeking the derivative, and its unique ideas and methods provide new possibilities for solving the problem of inequality persistence. By introducing Lopida's law, we can transform the complex inequality problem into simpler derivative problems, so as to simplify the problem solving process and improve the problem solving efficiency. At the same time, the application of Lopida's law can also help students to better understand the nature and laws of the inequality, and improve their mathematical literacy and problem-solving ability.

2 Overview of the Lopida Law

Lopida law is a method of solving limits in higher mathematics, which is also the teaching focus of higher mathematics. It is a method to use derivatives. Limit problem is one of the most important contents in higher mathematics, and also the foundation of higher mathematics. Therefore, it is of great significance to master the law of learning higher mathematics.

Definition of the Lopida Law

Lopida's law, also known as Lopida's theorem, is a fundamental theorem in calculus that is mainly used to solve limit problems under certain conditions. It is particularly suitable for solving the infinitive limit of type or type. By applying Lopida's rule, the complex limit problems can be transformed into a relatively simple derivative calculation problem.

Definine type 1.1 [1] $0/0$

If the function $f(x)$ and $g(x)$ meet the following conditions: if $\lim_{x\to a} f(x)=0$ and $\lim_{x\to a} g(x)=0$, and in the core neighborhood of point A, $f(x)$ and $g(x)$ can be derived and $g'(x)$ =0, then

$$\lim_{x\to a}\frac{f'(x)}{g'(x)}=\lim_{x\to a}\frac{f(x)}{g(x)}=A$$

Definine type 1.2 [1] ∞/∞

If the function $f(x)$ and $g(x)$ meet the following conditions: if $\lim_{x\to a} f(x)=\infty$ and $\lim_{x\to a} g(x)=\infty$, and in the core neighborhood of point A, $f(x)$ and $g(x)$ can be derived and $g'(x)$ =0, then

$$\lim_{x\to a}\frac{f'(x)}{g'(x)}=\lim_{x\to a}\frac{f(x)}{g(x)}=A$$

Application conditions of the Lopida rule

For individual pending type limits, their functions do not meet the conditions of using Lopida's law, and naturally cannot be applied, otherwise it may cause errors or no results. In order to use the law correctly, we should pay attention to the following problems.

(1) Before each use of Lopida's rule, it must be verified to be "" and "" type limits. Otherwise it causes an error.

(2) The law law is to guide the molecule and the denominator respectively, rather than the whole fraction.

(3) If the result obtained by Lopida's law is the real number or (no matter how many times are used), then the result of the original limit is the real number or; if the final limit does not exist (not), it cannot be asserted that the original limit does not exist, [1] should be considered to use other methods.

(4) It can be reused under the conditions of satisfying Lopida's law and theorem.

(5) The sequence limit can not be directly solved by Lopida's law to solve the [2].

Other infinities applying the Lopida rule

The inequality limit is also available $0\bullet\infty$, $\infty-\infty$, 1^{∞}, 0^{0}, ∞^{0} And so on type. After a simple transformation, they can generally be reduced to $0/0$ model or ∞/∞ model.

(1) $0\bullet\infty$ The infinitesimal or infinity in the product can be transformed onto the denominator $0/0$ or ∞/∞.

Example 1.1 $\lim_{x\to 0^+} x\ln x$.

Solution of the original=

$$\lim_{x\to 0^+}\frac{\ln x}{1/x}=\lim_{x\to 0^+}\frac{1/x}{-1/x^2}=\lim_{x\to 0^+}(-x)=0.$$

(2) $\infty-\infty$ Deform two infinity into two infinitesimal reversals and then divide them $0/0$.

Example 1.2 $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$.

Solution of the original=

$$\lim_{x\to 1}\frac{\ln x-(x-1)}{(x-1)\ln x}=\lim_{x\to 1}\frac{\frac{1}{x}-1}{\ln x-\frac{x-1}{x}}=\lim_{x\to 1}\frac{-\frac{1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}}=\lim_{x\to 1}\frac{-1}{x+1}=$$

(3) 1^{∞} Avitable log properties $e^{\ln a}=a$ Simplify the function to e Base number of the exponential function, to determine the limit of the exponential. The change method is used as follows

$$1^{\infty}=e^{\ln 1^{\infty}}=e^{\infty\bullet\ln 1}=e^{0\bullet\infty}.$$

At the same time, the equivalent infinitesimal size can also be used for different problems $\ln(1+x)\sim x$ Make a replacement, change the simple formula.

Example 1.3 $\lim_{x\to\infty}\left(\frac{x^2-1}{x^2+1}\right)^{x}$.

Solution of the original=

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$$\lim_{x\to\infty} e^{\ln(\frac{x^2-1}{x^2+1})^x} = e^{\lim_{x\to\infty} x\ln(1-\frac{2}{x^2+1})} = e^{\lim_{x\to\infty}\frac{\frac{-2}{x^2+1}}{1/x}} =$$

$$e^{\lim_{x\to\infty}\frac{\frac{4x}{(x^2+1)^2}}{(-1)/x^2}} = e^{\lim_{x\to\infty}\frac{-4x^3}{x^4+2x^2+1}} = 1 .$$

In the process of solving, the equivalent infinitesimal substitution, namely the handle $\ln(1-\frac{2}{x^2+1})$ Replace into $\frac{-2}{x^2+1}$.

(4) 0^0 The simplified method is as follows:

$$0^0 = e^{\ln 0^0} = e^{0\bullet\ln 0} = e^{0\bullet\infty} .$$

Example 1.4 $\lim_{x\to 0} x^{\frac{1}{1+\ln x}}$.

Solution of the original=

$$\lim_{x\to 0} e^{\ln x\frac{1}{1+\ln x}} = e^{\lim_{x\to 0}\ln x\frac{1}{1+\ln x}} = e^{\lim_{x\to 0}\frac{\ln x}{1+\ln x}} = e^{\lim_{x\to 0}\frac{1/x}{1/x}}$$

(5) ∞^0 The change method is as follows:

$$\infty^0 = e^{\ln\infty^0} = e^{0\bullet\ln\infty} = e^{0\bullet\infty} .$$

Example 1.5 $\lim_{x\to\infty}(1+x)^{\frac{1}{x}}$.

Solution of the original =

$$\lim_{x\to\infty} e^{\ln(1+x)^{\frac{1}{x}}} = e^{\lim_{x\to\infty}\frac{\ln(1+x)}{x}} = e^{\lim_{x\to\infty}\frac{1/(x+1)}{1}} = e^{\lim_{x\to\infty}(\frac{1}{1+}}$$

Note You cannot directly use the Lopida's rule in sequence cases because for discrete variables $n\in N^+$ You cannot find the derivative.

1.4 Lopida law uses uses failure cases

Although the Lopida law applies to solving the limit class problems, we often encounter some situations that are not suitable for using the Lopida rule. Here are some of the special cases where the Lopida rule fails.

(1) The limit does not exist after using Lopida's rule (not ∞) fluctuation.

Example 1.6

$$\lim_{x\to\infty}\frac{x+\sin x}{x} = \lim_{x\to\infty}\frac{1-\cos x}{1} .$$

This functional limit does not exist and does not satisfy the law " $\lim_{x\to a} f(x)/g(x) = \lim_{x\to a} f'(x)/g'(x)$ "

But it cannot be said that the original function limit does not exist.

$$\lim_{x\to\infty}\frac{x+\sin x}{x} = \lim_{x\to\infty} 1 + \lim_{x\to\infty}\frac{\sin x}{x} = 1 .$$

(2) Using Lopida's rule is becoming more and more complex to determine whether a limit exists.

Example1.8

$$\lim_{x\to 0}\frac{e^{-\frac{1}{x}}}{x^{50}} = \lim_{x\to 0}\frac{-e^{-\frac{1}{x}}\frac{1}{x^2}}{50x^{49}} = \lim_{x\to 0}\frac{-e^{-\frac{1}{x}}}{50x^{51}} .$$

Through the use of the Lopida law, we can see that the law will only make the molecule and the denominator more and more complex, and can not solve the limit, then the law fails.

solution for $t=\frac{1}{x}$, Then the original function

$$= \lim_{t\to\infty}\frac{e^{-t}}{t^{-50}} = \lim_{t\to\infty}\frac{t^{50}}{e^t} = 0 .$$

(3) After the zero.

Example 1.9

$$\lim_{x\to\infty}\frac{e^{2x}(\cos x+2\sin x)+e^{-2x}\sin^2 x}{e^x\sin x} .$$

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Take $x = n\pi - \frac{\pi}{4} \to \infty (n \to \infty)$, At this point, the derivative of the denominator has a zero, so the limit does not exist.

2 The type of high school inequality persistence problem

In high school mathematics, the problem of inequality persistence mainly includes linear inequality, quadratic inequality, parameter inequality and inequality group. These problems not only involve basic algebraic operations, but also require students to have certain mathematical thinking and logical reasoning ability.

2.1 linear inequality

Linear inequality, also known as a one-time function, can generally construct a function image and find the maximum value using monotonicity.

Theorem 2.1[3] For a primary function, $f(x) = kx + b, x \in [m, n]$,

$f(x) > 0$ was established

$\Leftrightarrow \begin{cases} f(m) > 0 \\ f(n) > 0 \end{cases}$; $f(x) < 0$ was established

$\Leftrightarrow \begin{cases} f(m) < 0 \\ f(n) < 0 \end{cases}$.

2.2 quadratic function inequality

Quadatic function type, in high school mathematics, quadratic function is very important, accounting for a very large proportion in the examination. Usually, the maximum value of the function is judged according to the opening direction of the quadratic function.

theorem2.2[3] If the quadratic inequality holds constant over the whole range of real numbers, it is solved by the following method:

take $f(x) = ax^2 + bx + c (a \neq 0)$, so

(1) $f(x) > 0$ in $x \in R$ was established

$\Leftrightarrow$ $a > 0$, and $\Delta < 0$;

(2) $f(x) < 0$ in $x \in R$ was established

$\Leftrightarrow$ $a < 0$, and $\Delta < 0$.

theorem2.3[3] If the quadratic inequality holds constant in a given interval, it is solved by using the distribution of the root:

quadratic function $f(x) = ax^2 + bx + c > 0 (a > 0)$ in $[m, n]$ was founded,There are

$$\begin{cases} f(m) > 0 \\ -\frac{b}{2a} < m \end{cases} \text{or} \begin{cases} m \le -\frac{b}{2a} \le n \\ \Delta = b^2 - 4ac < 0 \end{cases} \text{or}$$

$$\begin{cases} -\frac{b}{2a} > n \\ f(n) > 0 \end{cases} \Leftrightarrow f(x)_{\max} > 0, x \in [m, n].$$

2.3 Parametric inequalities are included

The parameter inequality is generally solved by separating variables. If two variables appear in the equation or inequality, the range of one variable is known, the scope of the other variable is unknown, and it is easy to classify the two variables into the equal or unequal sign, the constant establishment problem can be transformed into the most value problem.

theorem2.4[4] $a \ge f(x)$ the necessary

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and sufficient conditions for constant establishment are : $a \geq f(x)_{max}$; $a \leq f(x)$ the necessary and sufficient conditions for constant establishment are: $a \leq f(x)_{min}$. $f(x) > a$ in $x \in D$ the necessary and sufficient conditions for a solution are : $f(x)_{max} > a$; $f(x) < a$ in $x \in D$ the necessary and sufficient conditions for a solution are: $f(x)_{min} < a$.

Example 2.1 known function $f(x) = \begin{cases} -x^2 + x (x \leq 1) \\ \log_{1/3}^{x} (x > 1) \end{cases}$ If to any $x \in R$, inequality $f(x) \leq m^2 - \frac{3}{4}m$ was established,Seek real number m the range of values.

solution ①when $x \leq 1$,

$f(x) = -x^2 + x = -(x - \frac{1}{2})^2 + \frac{1}{4} \leq \frac{1}{4}$.

②when $x > 1$,

$f(x) = \log_{1/3}^{x} (\downarrow)$, $f(x) < 0$,so

$f(x)_{max} = \frac{1}{4}$, so $m^2 - \frac{3}{4}m \geq \frac{1}{4}$ was established,that $4m^2 - 3m - 1 \geq 0$, then

$(m-1)(4m+1) \geq 0 \Rightarrow m \leq -\frac{1}{4}$ or $m \geq 1$.

All in all, $m \in (-\infty, -\frac{1}{4}] \cup [1, +\infty)$.

2.4 Content word inequality

"Content word inequality problem" more in the exam of the exam, sometimes appear in the choice, it often with "arbitrary", "existence" words, this is a kind of contains full quantifier and comprehensive quantifier, intended to examine the logic and function of the basic knowledge, and the ability to deal with problems. Considering the problem of persistence from the perspective of functional properties, we have:

$\forall x_1, x_2 \in I, |f(x_1) - f(x_2)| \leq a$ was established $\Leftrightarrow$ $f(x)_{max} - f(x)_{min} \leq a$ in I was established;

$\forall x_1, x_2 \in I, |f(x_1) - f(x_2)| \geq a$ was established $\Leftrightarrow$ $f(x)_{max} - f(x)_{min} \geq a$ in I was established.

$\forall x_1 \in A, \forall x_2 \in B$, $f(x_1) \geq g(x_2)$ was established $\Leftrightarrow$ $f(x)_{min} \geq g(x)_{max}$ was established;

$\forall x_1 \in A, \forall x_2 \in B$, $f(x_1) \leq g(x_2)$ was established $\Leftrightarrow$ $f(x)_{max} \leq g(x)_{min}$ was established.

Example 2.2 known function $f(x) = \ln x - ax + \frac{1-a}{x} - 1 (a \in R)$. take $g(x) = x^2 - 2bx + 4$, when $a = \frac{1}{4}$,if any $x_1 \in (0,2)$, have $x_2 \in [1,2]$, make $f(x_1) \geq g(x_2)$, Seek real number b the range of values.

solution when $a = \frac{1}{4}$,

$f(x)=\ln x-\frac{1}{4}x+\frac{3}{4x}-1,$

$f'(x)=-\frac{(x-1)(x-3)}{4x^2}$, so $f(x)$ in $(0,1)$ is the minus function,in $(1,2)$ is increasing function , So for any $x_1\in(0,2)$, have

$f(x_1)\geq f(1)=-\frac{1}{2}.$

There is $x_2\in[1,2]$, take $f(x_1)\geq g(x_2)$,

so $-\frac{1}{2}\geq g(x_2)$, so $x\in[1,2]$, take

$g(x)=x^2-2bx+4\leq-\frac{1}{2}$, $2bx\geq x^2+\frac{9}{2}$,

$2b\geq x+\frac{9}{2x}\in[\frac{17}{4},\frac{11}{2}]$, so $2b\geq\frac{17}{4}$,

solution $b\geq\frac{17}{8}$, so b the range of values $[\frac{17}{8},+\infty)$.

Example 2.3function

$f_n(x)=x^n+bx+c(n\in N^*,b,c\in R)$.

take $n=2$, if any $x_1,x_2\in[-1,1]$, have $|f_2(x_1)-f_2(x_2)|\leq 4$, The range of values of b.

solution when $n=2$,

$f_2(x)=x^2+bx+c=(x+\frac{b}{2})^2+c-\frac{b^2}{4}.$

For any $x_1,x_2\in[-1,1]$, there are

$|f_2(x_1)-f_2(x_2)|\leq 4\Rightarrow|f_2(x_1)-f_2(x_2)|_{max}\leq 4,$

$f_2(x)$ in $[-1,1]$ the difference between the maximum and the minimum value of $M\leq 4$.

when $b<-2$, $f_2(x)$ in $[-1,1]$ decrease,

$M=f_2(-1)-f_2(1)=(1-b+c)-(1+b+c)=-2b\leq 4,$

so $b\geq -2$, rejection.

when $-2\leq b<0$, $f_2(x)$ in $[-1,-\frac{b}{2}]$ decrease,in $[-\frac{b}{2},1]$ increase,

$M=f(-1)-f(-\frac{b}{2})=(1-b+c)-(c-\frac{b^2}{4})=(\frac{b}{2}-1)^2\leq 4,$

solution $-2\leq b\leq 6$, so $-2\leq b<0$.

when $0\leq b\leq 2$, $f_2(x)$ in $[-1,-\frac{b}{2}]$ 上 increase,

$M=f(1)-f(-\frac{b}{2})=(1+b+c)-(c-\frac{b^2}{4})=(\frac{b}{2}+1)^2\leq 4,$

solution $-6\leq b\leq 2$, so $0\leq b\leq 2$.

when $b>2$, $f_2(x)$ in $[-1,1]$ 上 increase,

$M=f_2(1)-f_2(-1)=(1+b+c)-(1-b+c)=2b\leq 4,$

so $b\leq 2$, Conflict, give up.

to sum up,Knowable value b range $[-2,2]$.

3 Application of law in high school inequality was established problem

3.1 The application of Lopida's rule to the problem of was established including parameter inequality

To find the parameter value range of the function inequality was established, students often use the method of classification discussion and false take counterproof, but it is difficult to discuss the parameters. If the method of separating parameters and variables is adopted, it will be some trouble to find the maximum value (value domain) of the function after separation, such as the maximum value, extreme value in meaningless point, or tend to be infinite. This is better to handle [5]. The common types and true questions of fraction functions after separation parameters are as follows:

(1) Function after the separation parametersis $0/0$ (∞/∞)

Find the fractional function after the separation parameter $f(x)/g(x)$ limit value, Need to observe $x \to a(\infty)$, $f(x)$, $g(x)$ is tend to $0(\infty)$; if $f(x)$, $g(x)$ all tend to ∞, There are variations replaced by $1/f(x), 1/g(x)$ tend to 0, This istake uses one of the conditions of the law opLlaw.With the judgment $x \to a(\infty)$, $f'(x)/g'(x)$ Whether tends to constant A; if is, so A is $f(x)/g(x)$ limit value.To judge the monotonicity of the function, it is necessary to analyze the part of the formula in the derivative function again.

example3.1 (2022 National-22) known function $f(x)=xe^{ax}-e^{x}$, when $x>0$, $f(x)<-1$,

The range of values of a.

solution when $x=0$, $f(x)=0,\ a\in R$;

$f(x)<-1$

$$xe^{ax}-e^{x}+1<0 \Rightarrow e^{ax}<\frac{e^{x}-1}{x}$$

$$\Rightarrow a<\frac{\ln\frac{e^{x}-1}{x}}{x}.$$

take $h(x)=\dfrac{\ln\frac{e^{x}-1}{x}}{x}$, so $h'(x)=\dfrac{xe^{x}-e^{x}+1}{x^{3}(e^{x}-1)}$; take $\varphi(x)=xe^{x}-e^{x}+1$, so $\varphi'(x)=xe^{x}>0$ was established,so $\varphi(x)$ in $(0,+\infty)$ 上单调 increase,so $\varphi(x)>\varphi(0)=0$.

$\phi(x)=x^{3}(e^{x}-1)>0$, so $h'(x)>0$ in $x\in(0,+\infty)$ was established,

so $h(x)$ in $x\in(0,+\infty)$ increase,so

$$\lim_{x\to 0}h(x)=\lim_{x\to 0}\frac{\ln\frac{e^{x}-1}{x}}{x}=\lim_{x\to 0}\frac{xe^{x}-e^{x}+1}{x(e^{x}-1)}=\lim_{x\to 0}\frac{xe^{x}}{xe^{x}+e^{x}-1}$$

$$=\lim_{x\to 0}\frac{e^{x}+xe^{x}}{2e^{x}+xe^{x}}=\frac{1}{2},$$

so $x>0$, $h(x)>\frac{1}{2}$, so $a<\frac{1}{2}$.

example3.2[6] known function

$f(x)=\begin{cases}(2x-x^2)e^x, x\le 0,\\ -x^2+4x+3, x>0,\end{cases}$ $g(x)=f(x)+2$

If the function $g(x)$ are just two different zeros, to find the value range of the real number.

solution $y=(2x-x^2)e^x(x\le 0)$, Seek to guide $y'=(2-x^2)e^x$, so $y=(2x-x^2)e^x(x\le 0)$ in $(-\sqrt{2},0)$ increase,in $(-\infty,-\sqrt{2})$ decrease,when $x=-\sqrt{2}$,

$$y_{\min}=f(-\sqrt{2})=\frac{-(2+2\sqrt{2})}{e^{\sqrt{2}}},$$

Then by Lobidha so:

$$\lim_{x\to-\infty}(2x-x^2)e^x=\lim_{x\to-\infty}\frac{2x-x^2}{e^{-x}}=\lim_{x\to-\infty}\frac{2-2}{-e}$$

$y=-x^2+4x+3(x>0)$ in $(0,2)$ increase,in $(2,+\infty)$ decrease,so take function $g(x)$ are just two different zero points, $-2k=0$ or $\frac{-2\sqrt{2}-2}{e^{\sqrt{2}}}$ or $3<-2k<7$, the real number k range of values is $(-\frac{7}{2},-\frac{3}{2})\cup\left\{0,\frac{\sqrt{2}+1}{e^{\sqrt{2}}}\right\}$.

(2) After separating the parameters function is $0\bullet\infty$

In the parameter separation, find its limit and find itis " $0\bullet\infty$ ", This can divide the product into a form, is $0/0$ or ∞/∞ function ,

Find the limit value.

example3.3[5] take function $f(x)=\ln(x+1)+ae^{-x}-a, a\in R$. if $x\in(0,+\infty)$, $f(x)\ge 0$, The range of a values.

solution take $f(x)\ge 0$, to $x\in(0,+\infty)$ was established

$\Leftrightarrow a\le\frac{\ln(x+1)}{1-e^{-x}}=\frac{e^x}{e^x-1}\ln(x+1)$ to $x\in(0,+\infty)$ was established.

take $h(x)=\frac{e^x}{e^x-1}\ln(x+1)$, so

$$h'(x)=\frac{e^x[\frac{e^x-1}{x+1}-\ln(x+1)]}{(e^x-1)^2},$$ take

$\varphi(x)=\frac{e^x-1}{x+1}-\ln(x+1)$, so

$$\varphi'(x)=\frac{x(e^x-1)}{(x+1)^2}.$$

because $x>0$, $\varphi'(x)>0$, so $\varphi(x)$ in $(0,+\infty)$ increase,because $\varphi(x)>\varphi(0)=0$, so $x\in(0,+\infty)$, $h'(x)>0$, ao $h(x)$ in $(0,+\infty)$ increase.

when $x\to 0$, $\ln(x+1)\to 0$, $\frac{e^x}{e^x-1}\to\infty$,

$h(x)$ is " $0\bullet\infty$ " ,now take $h(x)$ product of the form of division $h(x)=\dfrac{\ln(x+1)}{1-e^{-x}}$,

$x\to 0$, $\ln(x+1)\to 0, 1-e^{-x}\to 0$, Meet the Lo Bida methodso,so

$$\lim_{x\to 0} h(x)=\lim_{x\to 0}\frac{\ln(x+1)}{1-e^{-x}}=\lim_{x\to 0}\frac{1/(x+1)}{e^{-x}}=1,$$

so $h(x)>1$, so $a\le 1$.

attention for function To limit is " $0\bullet\infty$ " , It can be multiplied into a form of division , that $0/0$ or ∞/∞ function .and if function $h(x)=x\ln x$, $x\to 0^{+}$, $x\to 0$, $\ln x\to -\infty$, $h(x)$ is " $0\bullet\infty$ " , $\lim_{x\to 0^{+}} x\ln x$ transfer $\lim_{x\to 0^{+}}\dfrac{\ln x}{1/x}$ (∞/∞) the limit can be found, and the if is converted into $\lim_{x\to 0^{+}}\dfrac{x}{1/\ln x}$ ($0/0$) so can't find out.Therefore, the function deformation after separation should satisfy the Lopida fa so condition.

(3) After separating the parameters function is $\infty-\infty$

example3.4[5] known function $f(x)=x-\ln(x+1)$, if to any $x\in[0,+\infty)$, there are $f(x)\le kx^2$ establish,Find the minimum value of the real number k.

solution because $x=0$, $f(x)\le kx^2$ apparently established, $k\in R$. so for any $x\in[0,+\infty)$, $f(x)\le kx^2$ was established $\Leftrightarrow x\in(0,+\infty)$, $f(x)\le kx^2$ was established

$\Leftrightarrow x\in(0,+\infty)$, $k\ge\dfrac{f(x)}{x^2}=\dfrac{1}{x}-\dfrac{\ln(x+1)}{x^2}$ was established.

Take $h(x)=\dfrac{1}{x}-\dfrac{\ln(x+1)}{x^2}$,

so $h'(x)=\dfrac{-\dfrac{x^2+2x}{x+1}+2\ln(x+1)}{x^3}$.

Take $\varphi(x)=-\dfrac{x^2+2x}{x+1}+2\ln(x+1)$, so $\varphi'(x)=-\dfrac{x^2}{(x+1)^2}$, because $x>0$, so $\varphi'(x)<0$, so $\varphi(x)$ in $(0,+\infty)$ decrease, $\varphi(x)<\varphi(0)=0$, so, $h'(x)<0$, so $h(x)$ in $(0,+\infty)$ decrease,when $x\to 0$, $\dfrac{1}{x}\to\infty$, $\dfrac{\ln(x+1)}{x^2}\to\infty$, $h(x)$ is " $\infty-\infty$ " ,now take $h(x)$ reduction of fractions to a common denominator is $h(x)=\dfrac{x-\ln(x+1)}{x^2}$, when $x\to 0$, $x-\ln(x+1)\to 0$, $x^2\to 0$, $h(x)$ Lopida was eligible,so have

$$\lim_{x\to 0} h(x)=\lim_{x\to 0}\frac{x-\ln(x+1)}{x^2}=\lim_{x\to 0}\frac{1-\frac{1}{x+1}}{2x}=\lim_{x\to 0}\frac{\frac{1}{(x+1)^2}}{2}=\frac{1}{2}$$

so $h(x)<\dfrac{1}{2}$, so $k\ge\dfrac{1}{2}$, so k minimum

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value is $\frac{1}{2}$.

attention The limit discovery of function after this topic is in finding the separation parameter, $h(x)$ is " $\infty-\infty$ ", this can be converted by using the through division $0/0$ function .

To find the value range class problem of the parameter in the was established problem of solution function inequality, function must meet the conditions of lopida fa so application after requiring attention to separation parameters. Through the above college entrance examination example question solution method, can summarize the application of lobidida method sosolution skills and methods of this kind of question. Firstly, the parameter is separated from the variable to obtain the resulting function after separation; secondly, judge the monotonicity of the function containing the variable after the separation parameter (obtained by derivative); finally, the then variable tends to a certain value to determine the fractional after separation function is one of $0/0$ or ∞/∞ , and find out the limit value.

3.2 Application of was established

In seek solution contains "in quantifier" and "full quantifier" inequality was established problem, we found that this kind of problem means in test the basic knowledge of logic and function, as well as the ability to comprehensively analyze and deal with the problem. In this kind of problem, we often encounter multiple function or and multiple unknown variables, which is more complicated by the problem of task inequality was established solution.

So we can use the Lopida method so to simplify the solution process and facilitate the calculation.

(1)Single function univariate For a problem with only one function and one unknown variable, in to seek the value range of the solution unknown quantity, usually need to move items, get about x function $g(x)$.

example3.5[7] (2015Shandong Volume Science)

take $f(x)=\ln(x+1)+a(x^2-x)$, there $a\in R$, if $\forall x>0$, $f(x)\geq 0$ establish,The range of values of a.

solution when $x=1$,It is known from the meaning of the question

$f(x)\geq 0$ was established.令

$g(x)=\frac{\ln(x+1)}{x-x^2}$, so

$$g'(x)=\frac{(x-x^2)+(2x^2+x-1)\ln(x+1)}{(x+1)(x-x^2)^2}.$$

Then make

$h(x)=(x-x^2)+(2x^2+x-1)\ln(x+1)$,

so $h'(x)=(4x+1)\ln(x+1)$.

when $x\in(0,1)$, $h'(x)>0$, so $h(x)$ in $(0,1)$

increase,so $h(x)>h(0)=0$, so

$g'(x)>0$, so $g(x)=\frac{\ln(x+1)}{x-x^2}$ in $(0,1)$ increase.

when $x\to 0^+$,there

$\ln(x+1) \to 0$, $x-x^2 \to 0$. By Lo Bida law

so

$$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \frac{\ln(x+1)}{x-x^2} = \lim_{x\to 0^+} \frac{\frac{1}{x+1}}{1-2x} = \frac{\lim_{x\to 0^+} \frac{1}{x+1}}{\lim_{x\to 0^+}(1-2x)} = 1,$$

So when $x \to 0^+$, $g(x) \to 1$, so $a \le 1$.

when $x \in (1,+\infty)$,similarly,

$g(x) = \dfrac{\ln(x+1)}{x-x^2}$ in $(1,+\infty)$ increase.

when $x \to +\infty$, $|\ln(x+1)| \to \infty$,

$|x-x^2| \to \infty$, By Lopida fa so have

$$\lim_{x\to +\infty} g(x) = \lim_{x\to +\infty} \frac{\ln(x+1)}{x-x^2} = \lim_{x\to +\infty} \frac{\frac{1}{x+1}}{1-2x} = - -$$

sowhen $x \to +\infty$, $g(x) \to 0$, so $a \ge 0$.

to sum up, a value range is $[0,1]$.

(2) The single function bivariates find solution around a single function, but in addition to the unknown number of the value range of solution, there are two variables. You need to find solution by taking the most value of a single variable.

example3.6[8] known number

$f(x) = \ln(\frac{1}{2}+\frac{1}{2}ax) + x^2 - ax$ (a is constant, $a > 0$) ,if for any $a \in (1,2)$,

always in $x_0 \in [\frac{1}{2},+\infty)$ take inequality

$f(x_0) > m(1-a^2)$ establish,take m the range of values.

solution when $a \in (1,2)$, $f(x)$ Seek guidance know,

$f(x)$ in $[\frac{1}{2},+\infty)$ maximum value on is

$$f(x)_{max} = f(1) = \ln(\frac{1}{2}+\frac{1}{2}a) + 1 - a.$$

The for-is problem is equivalent to: for the arbitrary $a \in (1,2)$,

$\ln(\frac{1}{2}+\frac{1}{2}a) + 1 - a + m(a^2-1) > 0$ was established.

So have

$m > \dfrac{\ln(\frac{1}{2}+\frac{1}{2}a)+1-a}{1-a^2}$, take

$g(a) = \dfrac{\ln(\frac{1}{2}+\frac{a}{2})+1-a}{1-a^2}$, By Lo Bida law

so

$$\lim_{a\to 1} g(a) = \lim_{a\to 1} \frac{\ln(\frac{1}{2}+\frac{a}{2})+1-a}{1-a^2} = \lim_{a\to 1} \frac{\frac{1}{1+a}-1}{-2a} = \frac{1}{4}.$$

So have $m \ge \frac{1}{4}$.

(3) Double function univariate values

there are two function $f(x)$ and $g(x)$, However, the variables were all x And the defined domain is R, that double function Univariate problem,because function The variables and domain are the same, this type can be converted to single function single variable type, that is, the study object is newly constructed function $H(x)$; Then transform the problem into solution function according to the type of inequality $H(x)$ Min in the domain;

Establish unequal relationships about a;

Find the solution range of a.

example3.7[5] (2014Shaanxi theory-21) take function $f(x)=\ln(x+1)$, $g(x)=xf'(x)$, $x\ge 0$, that $f'(x)$ is $f(x)$ Of the guide function ,if $f(x)\ge ag(x)$ was established,Find the value range of the real number a.

solution because $g(x)=\dfrac{x}{1+x}$, By the meaning of the question, $f(x)\ge ag(x)$ to $x\ge 0$ was established.

when $x=0$, $f(0)\ge ag(0)$, $a\in R$, The so upper formula can be launched $f(x)\ge ag(x)$ to $x>0$ was established, $a\le\dfrac{f(x)}{g(x)}=\dfrac{(x+1)\ln(x+1)}{x}$ to $x>0$ was established,take $h(x)=\dfrac{(x+1)\ln(x+1)}{x}$,

so $h'(x)=\dfrac{x-\ln(x+1)}{x^2}$, take $\varphi(x)=x-\ln(x+1)$, so $\varphi'(x)=\dfrac{x}{x+1}$.

when $x>0$, $\varphi'(x)>0$, so $\varphi(x)$ in $(0,+\infty)$ increase, $\varphi(x)>\varphi(0)=0$, so $h'(x)>0$, so that $h(x)$ in $(0,+\infty)$ increase;

when $x\to 0$, $(x+1)\ln(x+1)\to 0$, $h(x)$ Meet the conditions of Lopida fa so,so

$$\lim_{x\to 0} h(x)=\lim_{x\to 0}\frac{(x+}{}$$

sowhen $x>0$, $h(x)>1$, because $a\le h(x)$ $x>0$ was established,so $a\le 1$.

Conclusion

Through further study and analysis of the application of the law in the problem of high school inequality, we find that the law provides effective tools and methods for solving such problems. It not only simplifies the process of solving problems, improves the efficiency of solving problems, but also cultivates students' mathematical thinking and logical reasoning ability. At the same time, the results of the research also prove that the introduction of the law in high school mathematics teaching is effective and can significantly improve students' problem-solving ability and achievement.

However, there are still some limitations and challenges in the application of Lopida law in high school mathematics. For example, how to better combine the law of the law with other mathematical knowledge to form a complete knowledge system; how to make appropriate teaching plans and strategies according to the actual situation and cognitive level of students; how to further expand the application of the law to solve more types of mathematical problems. These are questions that need to be explored in future research.

Reference

[1] Chen Xuebing. Application of Lopida Law in Gaokao Mathematics [J]. Sino-Foreign Exchange, 2017 (21): 225.

[2] Sun Qiaoge. Some thoughts on the law of Lopida [J]. Scientific consultation, 2021 (10): 96-97.

[3] Wang Xun. A common solution to the problem of inequality persistence [J]. Mathematics, Physics and Chemistry for middle school students (academic research edition), 2012 (10): 14-14.

[4] Wang Linjun. Including parameter inequality constant establishment and solution problem [J]. Chinese External

Education (Basic Education Edition), 2013 (1): 54.

[5] Li Min. The application of Lopida law in the persistence of functional inequality [J]. Mathematics of Shanghai Middle School, 2015 (11): 20-22.

[6] Hu Yirong. Use Lopida's law to solve the function including reference problem [J]. Middle School Mathematics Research, 2017 (9): 36-38.

[7] Yang Gang. Use lopida law to solve the "constant establishment when the parameter value range" type of college entrance examination final exam [J]. Mathematics Teaching Newsletter, 2018 (12): 79-80.

[8] Men Tongyu, Wang Guili, Guo Lingxiao. Research on the practice of high school mathematics teaching to promote deep learning —— Take the problem of "including parameter inequality persistence" as an example [J]. Teaching research in Primary and Secondary schools, 2023,24 (3): 50-57.